A MODEL FOR PRODUCT LINE DECISIONS[1]

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MARKETING MIX DECISIONS AND

PRODUCT LINE CONSIDERATIONS

Most firms market a number of somewhat similar products or what has been called a product line. Product policies of diversification and new product introduction have been implemented by the widening of the product line. Depth in the product line has emerged within product areas as firms attempt to meet competition and satisfy the needs of subsegments of the market. Although the multiproduct firm has grown in importance, there has not been a corresponding growth in model building and research efforts to help solve the marketing problems of firms offering a line of goods. The purpose of this paper is to develop a theoretical product line model and present an emperical application of the model.

Product line decisions are difficult because the products in the line are not usually independent. They can not be individually optimized and then added to produce optimum product line results. The marketing mix established for one product may effect the sales of another product. This interdependency is the key consideration in product line decision making.

Two basic kinds of interdependencies exist _ _ complementarity and substitutability.[2] Substitutability implies an introspective consumer attitude that prescribes that one product will be substituted for the other under certain conditions. This substitutability may be monitored in the price-sales response between products. Economists

have defined the cross elasticity of price to measure this inter-
action. The cross price elasticity between products one and two is:

$$CP12 = \frac{dx_1/x_1}{dP_2/P_2}$$

$$x_1 = \text{sales of product 1}$$

$$P_2 = \text{price of product 2}$$

In general, if the cross price elasticity is positive, the products
are substitutes. This follows because if the price of product two
is decreased, the quanity of the product two sold generally increases,
and the quanity of product one decreases if product one and two are
substitutes.[3] This is in agreement with what a positive cross price
elasticity would predict. The converse of this argument establishes
that a negative cross price elasticity generally indicates complemen-
tarity. Price is not the only variable that may be used to monitor
interdependencies. Promotion might also be used. For example, one
could define a cross advertising elasticity as:

$$CA12 = \frac{dx_1/x_1}{dA_2/A_2}$$

$$x_1 = \text{sales of product one}$$

$$A_2 = \text{advertising for product two}$$

If this elasticity is positive, the goods demonstrate complementarity.
This is because, in general, if advertising for product two increases,
sales of product two increase and sales of product one increase if the
products are complementary. It is wise to monitor interdependencies
through several variables since a product may be a substitute with
respect to one and a complement with respect to another. These cross

elasticity concepts form a basis for a consideration of product
line interdependencies. Cross elasticities have been used in
other market research studies. Claycamp has used this approach
in measuring competitive interdependencies in retail gasoline rates.[4]
Massy and Frank estimated cross elasticities between time periods
for frequently purchased goods.[5] This paper's model is based on the
use of cross elasticities to measure the interdependencies between
products within the product line. The model developed in this paper
is a single period mathematical model of a given product line. The
variables of the model are the marketing mix elements for each pro-
duct in the line. The model is then optimized to produce the maximum
total product line profits.

This model should be especially useful to firms utilizing
brand managers, since it can serve as a basis of allocating resources
to each brand in the product line. The brand manager concept artifi-
cially imposes independence between specific products in the line by
delegating products to competing brand managers. But if resources
are allocated on the basis of product interdependencies at the market-
ing management level, the motivational advantages of the brand manager
concept and the use of product line resources to maximize total line
profits can be compatable.

A MATHEMATICAL MODEL OF THE PRODUCT LINE DECISION

The Model

The starting point for the mathematical product line model is
the representation of the demand for one product offered by the firm.

In general, let the demand for product "j" be as follows:

$$x_j = f(\text{reference sales forecast, industry marketing effects,}$$
$$\text{competitive effects, interaction effects})$$

The reference sales forecast represents the estimate of the demand

for the next time period given some level of price and non-price

variables by the firm and competitors. This estimate reflects

estimated aggregate economic and monetary conditions, as well as

specific industry and company predictions.

The combined effects of price and non-price marketing activ-

ity of all firms in an industry may cause a shift in the reference

sales forecast. This may be formulated as follows:[6]

(1)
$$\bar{x}_1 a \; P_{jI}^{EPI} \; A_{jI}^{EAI} \; D_{jI}^{EDI}$$

$\bar{x}_1$ = reference sales forecast for industry

a = scale constant

P_{jI} = industry price level for product j

A_{jI} = industry advertising level for product j

D_{jI} = industry distribution level for product j

EPI = industry price elasticity for product j

EAI = industry advertising elasticity for product j

EDI = industry distribution elasticity for product j

This expression implies that as industry price, advertising, or

distribution levels change, the total industry sales will change.

The elasticity reflects the proportionate changes in industry sales

resulting from a proportionate change in one of the variables.

The proportion of the total industry sales a member of the industry achieves may, as suggested by Philip Kotler, be represented by the ratio of the firm's marketing effectiveness to the total industry effectiveness.[7]

(2) market share for product j in firm one $= \dfrac{P_{1j}^{EP1}\, A_{1j}^{EA1}\, D_{1j}^{ED1}}{\sum\limits_{i} P_{ij}^{EPi}\, A_{ij}^{EAi}\, D_{ij}^{EDi}}$

P_{ij} = price of product j by firm i

A_{ij} = advertising level for product j by firm i

D_{ij} = distribution level for product j by firm i

EPi = price sensitivity for firm i and product j

EAi = competitive advertising sensitivity for firm i and product j

EDi = distribution sensitivity for firm i and product j

The sensitivities appear to be similar to elasticities, but they are not elasticities. They do not represent proportionate changes in market share as the result of proportionate changes in the variable. They do, however, represent the sensitivity of the market share to changes in the marketing mix for each firm. In (2) the sensitivities are subscripted to allow the possibility of differentiated products. Given this expression, the effects of various strategies and counter-strategies can be related to the share of market a firm will receive. For example, if firm one is the price leader for a homogeneous product market, a lowering of price by the firm would be followed by other firms with no change resulting in the share of market. Industry effects as described in the previous

section may be produced however. Given a strategy and set of

counter-strategies, this expression will predict market share

changes for changes in price and non-price variables.

Demand interdependencies may originate because of sub-

stitution or complementarity effects of one product with other

products offered by other firms. The interdependencies of product

M relative to product j can be expressed by:

(3) $\qquad b \ P_{IM}^{CPjM} \ A_{IM}^{CAjM} \ D_{IM}^{CDjM}$

b = scale constant

P_{IM} = price of product M by industry

A_{IM} = advertising level for product M by industry

D_{IM} = distribution level for product M by industry

$CPjM$ = cross price elasticity for product j and M

$CAjM$ = cross advertising elasticity for product j and M

$CDjM$ = cross distribution elasticity for product j and M

This expression estimates the changes in the sales of one product as

the result of changes in parameters of interrelated products offered

by the firm or other firms.

Combining these expressions the demand for product j is:

(4)
$$x_j = \bar{x}_j a \ P_{jI}^{EPI} \ A_{jI}^{EAI} \ D_{jI}^{EDI} \ [b \ \prod_M P_{IM}^{CPjM} \ A_{IM}^{CAjM} \ D_{IM}^{CDjM}] \ [\frac{P_{1j}^{EP1} \ A_{1j}^{EA1} \ D_{1j}^{ED1}}{\sum_i P_{ij}^{EPi} \ A_{ij}^{EAi} \ D_{ij}^{EDi}}]$$

where $\prod_M$ = product sum over M, M$\neq$j, and other notation as previously

defined.

Given constant direct and cross elasticities, this equation represents the demand for one product of a firm's product line. It should be noted that this formulation could be extended to include more than three marketing variables by the specification of the appropriate direct and cross elasticities.

The total revenue for the firm is:

$$TR = \sum_j P_j x_j$$

The variable cost of producing one of the firm's goods could be expressed as:

(5)
$$TVC_j = AVC_j (x_j) \prod_M (x_M)^{CCjM}$$

TVC_j = total variable cost of producing product j

AVC_j = average variable cost function for product j, if produced independently of other products

x_j = quantity of product j produced

x_M = quantity of product M produced, (M≠j)

$CCjM$ = cross cost elasticity of product j and M, (M≠j)

The total cost for the firm is:

$$TC = \sum_j TVC_j - FC$$

where FC = fixed costs. The profit is obviously total revenue less total costs.

The problem for the firm in the short run is to maximize the total profit subject to existing technical, managerial, financial and production constraints.

Determination of Optimum Marketing Mix for the Product Line

To find the optimum marketing mix for each product, the profit function specified in the model must be maximized. The profit function, however, does not possess nice convexity or concavity properties, so the utilization of standard non-linear programming routines is impossible.[8] Although the function could be theoretically maximized by Lagrangean analysis, the practical problem of solving the set of partial differential equations in this case eliminates it as a possible solution method.[9] It is also true that the optimum points will satisfy the Kuhn-Tucker conditions, but the location of these points for this function is not efficient with existing gradient methods. Developments in non-linear programming are possible and probable, so it may be anticipated that this function will bend under future analytical approaches.

The powerful characteristics of analytical solutions being unavailable, iterative trial and error search procedures appear to be a likely resort. A first approach would be to test an exhaustive set of discrete price and non-price levels for each product offered by the firm. With "n" products and price and advertising as variables this would indicate (K^{2n}) trials for the exhaustive set, where K = number of values of each variable to test. This could be a large set even for a relatively small number of products in the firm's product offering. For example, if ten values for the price and advertising variables for each of four products were to be tested in all combinations, the number of trials would be 10^8 or one hundred

million trials. Preliminary tests of this model indicate that a large size computer can run two thousand trials in one minute, so that in general, running the exhaustive set is outside practical consideration with the computer of today.

Compiling an exhaustive set may not be necessary. Search routines may be instituted to reduce the number of trials. For example, a two stage search procedure which first locates the rough optimum by trials of all combinations of the quartiles in the range and then tests all the combinations of the variables in best rough quartile range could reduce the calculation burden by a factor of about one thousand. With this search procedure the four product case could be tested on a ten item range for each variable in about fifty minutes on a high speed computer.

Effective search heuristics have been developed to aid in the search for solutions. These heuristics are in the form of rules that guide the direction of search. For example, the search may be guided by the slope of the profit surface at the point of the last evaluation. The search then moves in the direction of most rapid improvement. A number of such search techniques have been developed and each has been proven to be efficient for certain kinds of problems.[10]

Another approach to search heuristics has been made possible by on-line time shared computer systems. With on-line capability and an interactive program, a manager can control the search for the optimum. This man-machine interaction is especially important

in marketing problems since a marketing manager usually has a
supply of intuition that may efficiently be used to reduce
the search space. For example, the manager may have a good idea
of the range of prices he could consider. He may be more effective
than a mechanistic search heuristic in restricting the expenditure
of search resources and in generating meaningful decisions.

A simple interactive program would allow the manager to
specify an initial value for each marketing variable and a number
of values about these initial values. These values would be
tested and the resulting sales, market shares and profit infor-
mation would be presented to the manager at the on-line computer
console. He could then redefine the range to be tested and the
number of points to be tested in this range. This feedback of
information and respecification of the search area can be used to
telescope in upon a final value. After each set of trials has
been evaluated, the manager can specify a narrower range and
thereby approach the best values for the variables. This man-
machine interaction allows the intuitive judgment of the manager
to be utilized in defining an optimum marketing mix for the pro-
duct line.

Input Considerations

The feasibility of the application of this model rests upon
the ability to generate meaningful input, as well as on the presence
of a practical solution method. The direct and cross elasticities
could be estimated on a subjective basis that reflects the decision

makers best judgment. This approach might be justified since the decision must be made and if the model is not used, a much simpler and perhaps less accurate decision procedure would be used. Subjective inputs, however, should be used only after all empirical information relating to the problem has been considered.

Input for the demand model can be generated in part by statistical regressions on empirical data. The industry elasticities and cross elasticities could be estimated from a regression of industry sales on past industry price and non-price variables. The use of logarithms would make the regression linear and usual econometric procedures could be used to estimate the constants -- the elasticities and cross elasticities (see equation 1 and 3).

A similar procedure could not be used to determine firm competitive sensitivities since logarithms of the market share term would not produce a linear equation (see equation 2). The competitive price and non-price sensitivities could be estimated by an interactive search routine/to minimize the total variation between designed observed and predicted market shares[11]. The competitive input is completed with a formalization of the reaction functions of the competing firms. These might be obtained by examining the past competitive responses to price and non-price changes. The proposed model could be run for various strategies and counter strategies. The resulting payoff matrix could then be analyzed by Bayesian or game theory techniques.[12] Successive applications of the model for a number of future periods with the estimated reaction functions could yield valuable information concerning the

existence of an equilibrium, the rate of convergence to the
equilibrium, and the stability of the equilibrium, if it exists.

The cost cross elasticities could be approximated by exam-
ining the cost records of the firm for various quantity mixes or
by formulating a linear programming model to minimize the cost of
producing specified quantities of the firm's products. Successive
runs of the cost minimization model and a regression procedure
could yield estimates of the cost cross elasticities.

The examination of past data could be supplimented by
directed studies to measure the perceived interrelationships
between products. Such a procedure has been developed by Barnett
and Stefflre.[13] If this experimental procedure or the regression
approaches that have been outlined in this section are not practical
in specific situations, subjective estimates could make the model
workable.

AN EMPERICAL APPLICATION OF MODEL

A theoretical model such as the one developed in the previous
section does not become useful to marketing decision makers until
it has been shown to be workable in real world decision situations.
To test the applicability of the prepared model to a real product
line problem, emperical grocery store audit data for a line of re-
lated frequently purchased consumer goods were utilized.[14] These
product line data were used to estimate the parameters of the pro-
duct line model and an on-line computer search program was utilized
to derive the optimum marketing mix for each product in the producer's
line.

The Test Data: The data were obtained by two audits of each of

fifty stores in the *test market* ▇▇▇▇▇▇▇ area.   The second audit was

carried out four weeks after the first, and both audits were

carried out in each store on Friday or Saturday when weekly

shopping trips are common.   The audit recorded the price, deal,

shelf facings, and sales for each size of the product in the line

marketed by the producer and his competitors.

The product line contained three types of products.   All

three served the same food need, but were different in their

product features.   Product one was especially designed to have

a different taste.   Product two was the established form of the

product class and product three was modified in its ingredients

to have appeal to a special segment of the market.   Product one

was a new product and faced no competitors, while product two and

three were established food items and had many competitors.   The

producer who was viewed as making the product line decisions

offered one brand in product one's market, two brands in product

two's market, and one brand in product three's market.

The first problem to be faced in analyzing the data was

aggregation.   Statistical tests of the differences in the market

responses of the data showed the audits did not differ significantly

between weekends, so the two audits were combined and considered

as one set of observations.   The second aggregation problem was

related to defining the products to be considered in the line.

Since the data were collected for each *package* size, it would have been

conceivable to consider each size as a separate product.   This

would have produced a large number of products, so it was decided to
aggregate the package sizes for each brand produced by the firm.
The aggregation was based on calculating the average price per
unit and the weighted number of facings for the product. The
facings were weighted by the size of each package. The two brands
offered by the firm in the product two class were similarly
aggregated into one brand for the purposes of this test. The
number of competitors in the product two and three classes were
aggregated into one competitor in product two's market and one
competitor in product three's market The result of the aggregation
was a three product line which faced no competitors in product mar-
ket one, one competitor in product market two, and one competitor
in product market three.

　　　　For each product and competitor, the shelf price, number of
facings, deals, and special displays were recorded. Over ninety
percent of the data were recorded with some deal.   Ninty per-
cent of this was a "cents off" deal and it was reflected in the
shelf price values The remainder of the deals were largely bonus
size deals. These deals were reflected in the price when price per
unit was calculated. The apparent industry practice of dealing to express
price changes led to the removal of deals from consideration as a
separate variable. Special displays occurred so infrequently (less
than one percent of the data) that it was not considered as a vari-
able in the analysis.

　　　　The audits did not monitor national or local advertising
in the test area. It was assumed that none of the brands received

a disproportionate amount of local advertising at any of the audited
stores. National advertising was uniform across the area, so it
would not affect the differences between stores, although it might
affect the aggregate market response. The lack of advertising data
appeared to be tolerable given the audit data for the stores were
to be used to estimate product sales responses to parameters that
differed between the stores. It would have been interesting to
extend the analysis to advertising if additional data had been
available.

Model Parameter Estimation

Two separate estimation procedures were used. The industry
elasticities and cross elasticities were estimated by multiple
regressions of industry data for each product. The competitive sen-
sitivities were estimated by an interactive on-line search program
that minimized the total variation between the observed and pre-
dicted market share. Both these estimations were made on the as-
sumption that the almost one hundred store audits represented a simple
sample of sales response to the marketing variables. This assumption
requires that all the stores be similar with respect to the charac-
teristics that affect sales, except the price and facing variables.
The effect of store size was investigated and it was found that the
elasticity with respect to store size averaged .08 for the relevant
brands. This appeared reasonable and tended to support the assump-
tion that the stores were similar with respect to the market respon-
ses to the variables.

The industry elasticities and cross elasticities were obtained by regressions based on equations of the following form:

$$\ln(x_1) = \ln(a_1) + EPI_1 \ \ln(P_{1I}) + EFI_1 \ \ln(F_{1I})$$

$$+ \ CP12 \ \ln(P_{2I}) + CF12 \ \ln(F_{2I})$$

$$+ \ CP13 \ \ln(P_{3I}) + CF13 \ \ln(F_{3I})$$

x_1 = sales of product one

a_1 = constant

EPI_1 = industry price elasticity for product one

P_{jI} = industry price for product j

EFI_1 = industry facing elasticity for product one

F_{jI} = industry facings for product j

CPjM = cross price elasticity between product j and M, $j \neq M$

CFjM = cross facing elasticity between product j and M, $j \neq M$

Similar equations were utilized in regressions of product two and three. This is a linear equation and can be estimated by usual multiple linear regression procedures.

Before the regressions were run, the data were examined to see that the variation in price and facings between stores was wide enough to yield response estimates that would be useful in the model. In general, the standard deviation of prices was about ten percent of the average prices, and the standard deviation of facings was about twenty five percent of the average. These were deemed reasonable deviations for the analysis.

The elasticities and the associated t values are given in
table one along with the coefficients of determination and their
associated F values. All the direct price elasticities are nega-
tive as could be expected, and significant at least at the 5% level.
Product one is over twice as sensitive to price changes as products
two and three. The facings elasticities for products two and three
were positive and significant at the five percent level. The facing
elasticity for product one is negative. This does not agree with
the a priori feelings about the elasticities of promotional variables,
but it is small (.071) and so s its associated t statistic. The
cross elasticities of price and facings for product two with respect
to product one (CP12 and CF12) indicate product two is a complement
with respect to product one. This interdependency is symetric with
respect to price, since the cross price elasticity of the price of
product one with respect to product two is less than zero. It is
not symetric with respect to facings, since CF21 is negative, but
the value is small (-.09) as is its t statistic. Products two and
three display elasticities of complementarity, and the relationships
are symetric (CP23<0, CP32<0, CF23>0, and CF32>0). Product one and
three are asymetric, but only one of the four elasticities is signif-
icantly different from zero at the fifteen percent significance level.
This is CF13 and it is negative showing substitution with respect
to facings.

	Product One $j=1$	Product Two $j=2$	Product Three $j=3$
EPI_j	-3.525 (-3.79)*	-1.584 (-1.95)	-1.529 (-2.56)
EFI_j	- .071 (- .36)	.812 (3.9)	.350 (1.78)
CPj1		- .284 (- .30)	.861 (.78)
CFj1		- .09 (- .45)	.071 (.31)
CPj2	-1.332 (-1.69)		- .548 (- .59)
CFj2	.936 (4.75)		.475 (2.04)
CPj3	- .222 (- .43)	- .324 (- .62)	
CFj3	- .304 (-1.83)	.120 (.74)	
R^2	.448	.292	252
F(6,88)	11.9	6.0	4.9

CPiN>0 => Substitutes   CFiN>0 => Complements

CPiN<0 => Complements   CFiN<0 => Substitutes

* t statistic for regression coeficient (elasticity) for 88 degrees of freedom

TABLE ONE

INDUSTRY ELASTICITIES & CROSS ELASTICITIES

This set of cross elasticities was one predominated by complementarity. This conclusion did not agree with a priori feelings that indicated these products could be competing for the same buyers in the general product class. Studies by Henderson, Hind, and Brown had indicated, in particular, that display space interactions between the frequently purchased consumer products of apples, oranges, grapefruit and bananas were basically ones of substitution.[15] These two considerations suggested further study should be initiated. To explore the interdependencies more completely, the industry price and facing data for each product were subdivided into "us" and "them" classes. "Us" was our brand price or facings and "them" was the average price and facing for all other competitors. This increased the number of parameters to be estimated, but yielded valuable information. The elasticities and cross elasticities for the firm ("us") and the competitors ("them") in each product are indicated in table two. In this table the variables that are significant by the t statistic with 84 degrees of freedom at least at the fifteen percent level are indicated by an asterisk. An examination of these cross elasticities for product one indicates that product one is complementary to both our brand and other competitive brands of product two. Cross elasticities for product three indicate that product one is complementary to our brand of product three, but shows substitution effects with other brands in product three. This is a new interdependency not indicated in table one.

	Product One $j=1$	Product Two $j=2$	Product Three $j=3$
EPI_j – us – –them–	-3.83* Δ	-1.86* -1.01	-2.64* .26*
EFI_j – us – –them–	– .173*	.113 .837*	.157 .038
CPj1 – us –		.266	1.28
CFj1 – us –		– .081	.162
CPj2 – us – –them–	– .91* – .55		1.67* – .61
CFj2 – us – –them–	.24* .38*		– .105 .41*
CPj3 – us – –them–	– .29 .207*	-1.55* .136*	
CFj3 – us – –them–	.11 – .301*	– .42* .09	
R^2	.52	.61	.51
F(10,84)	9.1	13.1	8.8

    us = elasticity of firm ("us") in product i's market

  them = elasticity of competitors ("them") in product i's market

    * = significant at least at the 15% level – one tail for
        direct elasticities and two tail tests on cross elasticities

   Δ = there was no competition in product one's market

TABLE TWO

INDUSTRY ELASTICITIES AND CROSS ELASTICITIES
SEPARATED BY PRODUCER AND HIS COMPETITORS

Product two's direct price and facings elasticities indicate that our brand and competitive brands have about the same effect in changing industry sales, since the price and facing elasticities for "us" and "them" are similar. Product two's sales showed no significant interdependency with product one's price or facings, but product two was inter-related with product three. Product two was complementary to our brand in the product three market but was a substitute for competitive brands in product three's market. Although the industry regressions in table one showed products one and two to be complements, the new regression (table two) indicates the complementarity is with our brands of product three, and that product two is competitive with other brands of product three.

Product three's direct elasticities indicate that our brand has a much greater effect on increasing industry sales than other brands of product three. In fact, the direct elasticity for other brands is perverse. This may be due to the dominance of our brand's price in the market and the insensitivity of other brand's prices in this market in producing changes in industry sales of product three. Product three showed no significant interdependencies with product one, but displayed some interesting relationships with product two. $CP32 = 1.67$ and therefore indicates our brand of product two is a substitute for product three. But $CP23 = -1.55$ indicating product two's sales are complementary to our brand of product three. The asymetry in the interdependency is significant at the five percent level and the elasticities are large (i.e. greater than one.) This asymetry was difficult to accept, so stepwise multiple regressions

were run for our brand sales in product markets two and three.[16]
The asymetry again appeared at the five percent significance level.
The fact that product three felt substitution effects from our
brand of product two is understandable. As the price of our brand
of product two increases, product two buyers substitute product
three. The fact the product two feels complementary effects from
our brand of product three is more difficult to explain. CP23 = -1.55
so that a ten percent reduction in the price of our brand of product
three causes a 15.5% increase in the sales of product two. This
could occur if buyers of our brand of product three also bought
product two when they perceived "low" prices for this class of goods.
If the perception is based on the price of product three, lowering of
prices of our brand of product three could cause a perception of low
prices for these buyers and they might then buy more of product two
with little sensitivity to product two's price. This explanation
rests on the existance of a buying rule for regular product three
purchasers. This can not be fully accepted until further study has
been carried out, but the significance of the asymetry indicates
the study would probably be worthwhile. Another factor moving in
direction of complementarity is the income effect caused by lower
prices in brand three. This increase in real income might lead to
additional purchases of our brand of product two.

The new regressions based on "us" and "them" in each product
class yielded new information and suggested new questions about the
product interdependencies between our brands and competitive brands.

The analysis indicated that significant interdependencies existed
between the three products in the firm's line.

The estimation of the competitive sensitivities to be uti-
lized in describing market share affects (see equation 2) was
carried out by a computer program to minimize the total variation
between the market share predicted by equation two, given a set
of elasticities, and the observed market shares. The estimation
was executed on the MIT computation center compatable time sharing
system. The interactive ability of an on-line system was utilized
in a conversational program which asked the researcher or manager
to supply initial estimates of the price and facing sensitivities
for the firm and its competitors. These initial sensitivities are
incremented by a delta prescribed by the manager. The number of
incremental steps to be taken for each sensitivity is also an on-
line input supplied by the manager. All combinations of the initial
and incremented sensitivities are evaluated, and the set of sensitiv-
ities producing the minimum total variation for the audit data points
is recorded. After this iteration is complete, the manager is asked
to supply a new set of deltas and number of steps. The next evalua-
tion uses the best past estimates as initial values. By continuing
this process, the manager can guide the search until it has reached
a prescribed level of accuracy. This procedure does not quarantee
that the optimal fit has been achieved; rather it is a heuristic
procedure based on the manager's best judgment and the computational
power of a high speed computer.

The minimum variation estimates for the three product markets are shown in table three.

	Product Two		Product Three	
	Price	Facing	Price	Facing
Our Firm	- .27	1.31	- .855	1.13
Competitor	.00	.75	-1.24	1.20

TABLE 3

COMPETITIVE PRICE AND FACING SENSITIVITIES

The minimum variation estimation procedure required 45 minutes of IBM7094 time. The estimates and the associated equation (2) explained 24% of the variance of the market shares in product two and 54% of product three market shares.

The sensitivities in product two indicate that the competitor has little or no effect on market share by his change in prices while changes in our price have a negative sensitivity. The zero value for the sensitivity of the competitor's price in market two may reflect the aggregate definition of the competitor or the competitive strategy employed. The facings sensitivities indicate our firm's facings are 1.5 times as effective as the competitor in producing market share changes. The product three competitive estimates indicate our firm's prices and facing variables are less effective in changing market share than the competitor.

The estimation of the direct cross and competitive elastic-
ities completes the parameter estimation for the model's
demand equation (see equation 4.) The final demand input was the
strategy that competitors were expected to utilize. The competitors
were expected to be non-adaptive with respect to the number facings
and price for product two. Competitors were expected to set a price
of 16.0 and allocate 4.0 facings to product two. Competitors in
product three were expected to be followers. They were expected
to imitate the number of facings and follow our price, but at a
level 2.0 cents below our price. The remainder of the input
necessary for the model is the cost function (see equation 5.)
In this test application, the costs for each product were independ-
ent, and the marginal costs were considered constant over the mean-
ingful range of production.

Optimization of Product Line Profit

The maximization of the product line profit was carried out
by an on-line computer search routine. The trial and error routine
began by evaluating a reference set of price and facings for each
of the three products. It then examined a range of values on each
side of the reference values in discrete steps. The range and
steps were specified from the remote computer console. Given the
increments and ranges, all combinations of the trial values were
run, and the best total product line profit (see equations 1 to 5)
was recorded. After the first series of trials have been reported,
the manager can re-specify the step intervals and ranges. By contin-
uing this process he can achieve the desired level of accuracy.

In the optimization routine, only elasticities significant at
least at the .15 level were considered, and the search was bounded
by the range of the data used for estimation. One constraint was
placed on the search. This was that the total facings must be less
than eight for our product line. This constraint forces the search
routine to allocate the shelf space optimally between the products
in the line.

The search program results are shown in table four. The
first column in table four gives the price, facings, profit, sales
and market share for each product at the reference level. The
reference level in this case was the average price and facings for
the products.

The next question faced was whether the interdependencies
between products should be considered at the aggregate industry
level (see table 1), or split industry level (see table 2.) The
approach taken was to carry out a sensitivity analysis and deter-
mine if the optimum marketing strategy was different under varing
interaction assumptions. This analysis began by determining the
optimum program assuming no interdependencies, and then repeating
the analysis with industry and split industry interactions.

The optimum program with no interactions was considerably
different from the reference program. The price of product one was
decreased while the prices for product two and three were increased
The facings allocation was also altered. Products one and two re-
ceived fewer facings while product three received more.

	REFERENCE	NO INTERACTION	INDUSTRY INTERACTION	SPLIT INTERACTIONS	INDUSTRY INTERACTIONS	COST SENSITIVITY Industry Interactions
Price of Product 1	20.5	16.8	16.8	16.8	16.8	19.8
Price of Product 2	17.0	22.0 *	22.0 *	22.0 *	22.0 *	22.0 *
Price of Product 3	21.0	26.0 *	26.0 *	16.0 *	26.0 *	26.0 *
Facings of Product 1	2.00	.75*	.75*	.75*	.75*	.875
Facings of Product 2	4.00	3.50	6.25	6.50	6.25	4.375
Facings of Product 3	2.00	3.75	1.00	.75*	3.00	2.375
Total Profit	382600	594500	585180	908700	587300	576200
Profit for Product 1	107100	128000	158700	154000	107900	73000
Profit for Product 2	184600	299800	324400	704800	324500	361200
Profit for Product 3	90900	166700	102000	49900	154900	142000
Sale for Product 1	1500000	3083200	3722200	3625600	2665400	1603500
Sale for Product 2	2500000	2421700	2611100	5537100	2611100	2508300
Sale for Product 3	1000000	1167400	762200	1165200	1093400	1012700
Market Share for Prod. 1	100	100	100	100	100	100
Market Share for Prod. 2	29.64	28.76	31.01	31.1	31.01	29.70
Market Share for Prod. 3	74.79	70.03	72.03	80.37	70.37	70.70
Facing Limit	8	8	8	8	10	8

* These values are at the upper or lower limit of the data used for estimation

TABLE FOUR

SEARCH RESULTS

The result of this program was an improvement in profit of over
50% and an increase in the sales of each product. The market
shares of product two and three dropped. This emphasizes that
defending market share may be dangerous since it may be consid-
erablely more profitable to let market share slip under some
conditions.

The optimum program with aggregate industry interaction led
to the same price structure as the no interaction case. The fac-
ing allocation was changed, however (see table four.) The facings
were concentrated in product two. This was due to the complementarity
between the facings of product two and sales of products one and three,
and the substitution effect between product three's facings and product
one's sales (see table 1.) The total profit for this program was
less than with no interactions. The profit decrease took place in
product three, apparently because of the optimum reallocation of its
facing allotment produced by the considerations of product inter-
dependency. The new strategy led to an increase in the market share
of products two and three. The share of market for product three
increased because competitors in product three were assumed to be
followers with respect to facings and price. The inclusion of aggre-
gate industry interdependencies in the analysis restricted the
maximum product line profit and led to a reallocation of effort be-
tween the products.

The third phase of the sensitivity analysis was to determine
the maximum profit when split industry interactions were included.
The best policy in this case would utilize a different pricing

strategy. The price of product three is lower. In fact, it is even lower than the reference price. This is primarily due to the complementarity of the price of our brand of product three and the sales of product two (see table 2.) The facings allocation is similar to the results obtained by the consideration of aggregate industry interactions, but the concentration of facings in product two is slightly greater. The product line profits in this analysis are much greater than in the no interaction or industry interaction cases. Additional profits occur in product two, while the profits in product three decrease almost 70%. This occurs because of the product interdependencies between our brands of product two and three. The consideration of the split interactions produces important changes in pricing strategy and total line profits.

The optimization analysis indicated that changes in the product line pricing policy and facing allocations could induce large changes in the total line profitability. Sensitivity analysis indicated that the facing allocation should be concentrated in product two if product interdependencies are considered. The price of product three and total profits were sensitive to the splitting of interactions between our brands and competitor's brands. Without the split, the price of product three should be raised, while with the split it should be lowered.

Two more sensitivity runs were carried out for the industry interaction case. The first was to determine the effect of relaxing the facing constraint. The constraint was changed to ten facings and the optimization repeated. The profit increased $2220, and all

the additional facings were allocated to product three. If stores could be convinced to allow two more facings for our product line for a cost of less than $2220, profits could be increased by such an action. The second sensitivity run was based on changes in the production costs of the products. Increasing the costs of product one by 17% and decreasing the cost of products two and three by about 20% led to a higher price for product one, but no change in the price of products two and three. The cost changes also affected the facing allocations. The degree of concentration in product two was reduced, and product three facings were increased above the reference level. The marketing mix and product interdependency effects are apparently sensitive to cost changes. These complexities emphasize the need for a model to aid in product line decisions.

The output of the optimization can be summarized in the recommendations that the price of product one be lowered, the price of product two be raised, and the facings allocation be more concentrated on product two. The price of product three should be lowered if the asymetric interdependency between products two and three is real. Additional study is recommended to ascertain the underlying behavior that generates the interdependency. All these recommendations should lead to an inflow of new data and additional analysis. The feedback should be utilized to update the existing parameter estimates and generate new recommendations. A continuing experimental procedure and a full adaptive model design could be implemented to improve and update decisions.[17]

SUMMARY AND EXTENSION

This paper has presented a theoretical model for use in solving the problem of finding the optimum marketing mix for each product in an inter-related product line. The interdependencies between products were incorporated in cross elasticities that represent the proportionate changes in the sales of one product as a result of changes in another product's marketing parameters. The interactions along with competitive and industry effects were combined in a demand equation for each product. The demand relationships were based on constant elasticity response functions. It could be a useful extension of this analysis to develop estimation routines for more general and complex demand response functions.[18] The demand and cost equations were used to specify a single period profit model. The dynamic problems of carryover effects of marketing variables were not considered. The extension of this analysis to multiperiod market mix determination would be worthwhile. Input and solution questions associated with the proposed product line model were discussed,and estimation and optimization techniques appropriate to the model were demonstrated by an application of the decision procedure to a three product line. It would be useful to widen the test product definitions to include other classes of products or to narrow the product definition to consider package sizes in each product. A hierarchy of interdependencies exist,and a sequential application of the model to increasingly more specific product definitions would be appropriate.

The data for this study was based on one series of store audits, but it would be desirable to have an information system and to build a bank of data on the product's performance so that more accurate input estimates could be obtained. Multiple regressions of the audit data were used to produce estimates of industry direct and cross elasticities. Competitive sensitivities were determined by an on-line interactive program that minimized the total variation between observed and expected market share.

The parameter estimates, costs and strategy inputs were utilized in an interactive search program which maximized the total product line profit. Each search (see table 4) consumed about fifteen minutes of IBM7094 time. A useful extension of this analysis could be directed at increasing the search effeciency by heuristic programming while still maintaining the advantages of the manager-machine interaction. In the test application, the search routine improved the profits over 50% from the profit level associated with the existing policy. Specifically, consideration should be given to reducing the price of product one, increasing the price of product two, and concentrating the shelf space allocation on product two. Decreasing the price of product three is indicated, but this action will be dependent upon studies that determine if the asymetry in the interdependency between products two and three exists. If it does exist, the price of product three should be reduced.

A final extension to the analysis would be to analyze the effects of adding or dropping a product from the line.[19] The

area of product line decisions is a fertile and important area of research. It is hoped that this model will be the basis of continued research into the effects of product interdependency and how these effects can be managed to produce improved product line profits.

FOOTNOTES

[1] The author would like to acknowledge the data manipulation assistance provided by Roy Dorrance and Dick Chandler.

[2] See Alfred Oxenfeldt,"Product Line Pricing," _Harvard Business Review_, XLIV, July-August 1966, pp. 134-144, for a verbal discussion of product line interdependency.

[3] This reassuring is not valid for a product that violates the law of demand (e.g. a Giffin good) because then as price increases, its sales increase.

[4] H. J. Claycamp, "Dynamic Effects of Short Duration Price Differentials in Retail Gasoline Sales," _Journal of Marketing Research_, III, May 1966, pp. 175-178.

[5] William E. Massy and Ronald E. Frank, "Short Term Price and Dealing Effects in Selected Market Segments," _Journal of Marketing Research_, II, May 1965, pp. 171-185.

[6] This is similar in form to the Cobb-Douglas production function. See Paul H. Douglas, _Theory of Wages_ (New York: Macmillan 1934). See Philip Kotler, "Competitive Strategies for New Product Marketing Over the Life Cycle," _Management Science_ XII, December 1965, pp. 104-119.

[7] Kotler, ibid., p. 107.

[8] Even if the market share effects are omitted, the function is only simplified to a sum of convex and concave functions.

[9] See Bob R. Holden, _The Structure of a Retail Market and the Market Behavior of Retail Units_, (Prentice Hall: Englewood Cliffs, New Jersey, 1960) for a calculus formulation of a product line problem.

[10] See Douglass J. Wilde and Charles S. Beightler, _Foundations of Optimization_ (Prentice Hall, 1967) for an exhaustive discussion of these issues.

[11] See application section of this paper for further information.

[12] Kotler, op. cit.

[13] Norman L. Barnett and Volney J. Stefflre, "An Empirical Approach to the Development of Advertising New Products." Presented at the 1966 Fall Conference of the American Marketing Association, Bloomington, Indiana.

[14] The author would like to thank Market Research Corporation of America for the use of this data. The product line is not identified so as to protect the interests of the producer and MRCA.

[15] Peter L. Henderson, James F. Hind, and Sidney E. Brown, "Sales Effects of Two Campaign Themes," Journal of Advertising Research I, December 1961, p. 11.

[16] Analysis of the correlation matrix showed little multi-colinearity between the variables. Since the data was taken at one point in time auto-correlation in the data was not suspected. If there had been multicolinearity in auto-correlation, this might have caused the asymetry.

[17] See John D. C. Little, "A Model of Adaptive Control of Promotional Spending," Operations Research XIV, November-December 1966, pp. 175-197, and "A Multivariate Adaptive Control Model," Alfred P. Sloan School of Management, MIT, Working Paper 211-66.

[18] See Glen L. Urban, "A Quantitative Approach to New Product Decisions," Alfred P. Sloan School of Management, MIT, Working Paper 215-66 for an exposition of such general response functions.

[19] See Urban, op. cit., for an analysis of this problem.

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